TOPICS FOR MATH 162

(1) Integration
  - Partitions, upper sums, lower sums, upper integral, lower integral
  - Refinements of partitions, proof that $\underline{L}(f) \leq \underline{U}(f)$.
  - **Definition** integrable function, integral.
  - Example of a non-integrable function.
  - Proving (e.g.) x is integrable from the definition.
  - $\int_a^b f + g$, $\int_a^b f + \int_{b^c} f$.
  - "$U(f, P) - L(f, P)$" criterion for integrability.
  - **Theorem** a continuous function is integrable.

(2) Fundamental Theorem(s) of Calculus
  - Proof that $\int_a^x f$ is a continuous function of f if f integrable.
  - **Theorem** (the first fundamental theorem of calculus) $(\int_a^x f)' = f(x)$ if f cts at x. Proof from definition of derivative.
  - **Theorem** (the second fundamental theorem of calculus) if $F' = f$ is integrable, then
$$\int_a^b f = F(b) - F(a)$$
. (Easy) proof from FTC1 if f is continuous, (harder) proof from mean value theorem in general.

(3) Special functions
  - **definition** of log, exp, and proofs of basic properties.
  - Limits involving exp ('exponential beats polynomial') and log.
  - I will **not** test the definitions of trig functions.
  - Trig identities (addition formulae) and their proof.

(4) Techniques of integration
  - Parts.
  - Substitution.
  - Guessing.
  - Partial fractions.
  - Some integrals you should definitely know, or know how to do:
    - $\int x^n$, $\int \sin(x)$, $\int \cos(x)$, $\int \tan(x)$;
    - $\int \log(x)$; $\int xe^x$;
    - $\int \frac{f'}{f}$;
    - $\int \frac{1}{1+x^2}$, $\int \frac{1}{1-x^2}$, $\int \sqrt{1-x^2}$, $\int \frac{1}{\sqrt{1-x^2}}$;
    - $\int \sec(x)$.
  - Integrals similar to those in questions 1-5, of Spivak (but not as difficult as those in the 'no holds barred' problem).
  - Reduction formulae (you don't need to know the formulae, just the general idea/method).
  - Parametric curves and arc length (I will not give you the formula!); example of the cycloid.

- Volumes of revolution (I will give you the formulae).
- Improper integrals (you should know the definition as a limit, and determine rigorously whether they converge).

(5) Sequences
- **Definition** convergence of a sequence. Basic propositions about this.
- Determining convergence and limits.
- **Theorem** bounded increasing sequence convergence.
- **Theorem** Bolzano–Weierstrass theorem.
- **Definition** Cauchy sequence. **Theorem** Convergent iff Cauchy.

(6) Series
- **Definition** summable sequence (= convergent infinite series) in terms of partial sums.
- Harmonic series, geometric series.
- Boundedness criterion for sequences with non-negative terms; **comparison test**.
- **Ratio test**, **limit comparison test**, **integral test** and their proofs.
- Determining convergence of series with non-negative terms using these tests; some you should 'just know'.
- **Alternating series test.**
- **Definition** absolute convergence. Absolute convergence implies convergence.
- **Theorem** rearrangement of absolutely convergent series.
- Counterexample for rearrangement of conditionally convergent series (I won't test the proof of the theorem here).

(7) Taylor polynomials
- **Definition** of nth Taylor polynomial of f at a. $\sin, \cos, \log, \exp$.
- Remainder term.
- **Theorem** remainder term $R_{n,a}$ vanishes to order n at a.
- Example of e^{-1/x^2}.
- Taylor series for $\log(1+x)$, $\arctan(x)$ converge (for certain x).
- **Theorem** Taylor's theorem with integral form for remainder:

$$R_{n,a}(x) = \int_a^x \frac{f^{(n+1)}(t)}{n!}(x-t)^n \, dt.$$

- Application to sin, cos, exp. Approximating values of these functions to within given error.