MATH 159 – HOMEWORK 9

This homework is not graded. All vector spaces are finite dimensional over F, unless otherwise stated.

(1) (a) Find a 2×3 matrix A and a 3×2 matrix B such that $AB = I_2$.
  (b) Show that it's impossible to find a 2×3 matrix A and a 3×2 matrix B such that $BA = I_3$.

(2) Find the inverses of the following matrices over $\mathbb{R}$:
  (a) $\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$;
  (b) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$;
  (c) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$;
  (d) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 3 & 4 & 3 \end{pmatrix}$;
  (e) The $n \times n$ matrix with every entry below the diagonal being 0 and every entry above or on the diagonal being 1.

(3) (See question 2 from last time). Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be reflection in the line $y = 2x$. Write down the matrix of T with respect to the ordered basis $((1, 2), (-2, 1))$. Use the change of basis formula to find the matrix of T with respect to the standard basis of $\mathbb{R}^2$.

(4) Find all the eigenvalues and eigenvectors of the following matrices:
  (a) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$;
  (b) $\begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}$;
  (c) $\begin{pmatrix} -5 & 1 \\ 0 & 7 \end{pmatrix}$;
  (d) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

(5) (a) Show that, if $T : V \to V$ is a linear map that has three different eigenvalues $\lambda_1, \lambda_2, \lambda_3$ with corresponding eigenvectors v_1, v_2, v_3, then v_1, v_2 and v_3 are linearly independent. *Hint: suppose that there is a linear combination of the eigenvectors that makes zero, and apply T to it.*
  (b) Show by induction that, if $T : V \to V$ is a linear map with k distinct eigenvalues $\lambda_1, \ldots, \lambda_k$ with corresponding eigenvectors $v_1, \ldots, v_k$, then $v_1, \ldots, v_k$ are linearly independent.