MATH 159 – HOMEWORK 7

Due 2pm on May 24th. Submit all problems; only the starred problems are graded.

(1) (*) Use partial differentiation to find the derivatives of the following functions:
  (a) $f(x,y) = e^{x^2+2y}$;
  (b) $f(x,y,z) = x^{y+z}$;
  (c) $f(x,y) = \frac{xy}{x^2+y}$;
  (d) $f(x,y,z) = z^2 \tan(xy)$.

(2) (*) Suppose that f is a differentiable function $\mathbb{R}^2 \to \mathbb{R}$ and that g and h are differentiable functions $\mathbb{R} \to \mathbb{R}$. What are the partial derivatives of the function $F : \mathbb{R}^3 \to \mathbb{R}$ defined by $F(x,y,z) = f(g(x+y), h(yz))$ in terms of those of f, g and h (hint: use the chain rule)?

(3) (*) Let $f(x,y,z) = 3x^2 + 2y^2 + z^2$. Find an equation for the tangent plane to the surface $f(x,y,z) = 6$ at the point $(1,1,1)$.

(4) (*) Let $f(x,y) = \frac{x^2}{1-y^2}$. Draw the contours $f(x,y) = 1$, $f(x,y) = 0$, and $f(x,y) = -1$. Find an equation for the tangent plane to the surface $z = f(x,y)$ at $(3,2,-3)$.

(5) From the definition $D_u f(x) = Df(x)(\hat{u})$, prove that

$$D_u f(x) = \frac{d}{dt}\bigg|_{t=0} f(x + t\hat{u}).$$

(6) Let $f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$. Find the directional derivative of f at $(2,1,1)$ in the direction $(1,1,1)$.

(7) (*) Find all of the critical points of $f(x,y) = 3xy - x^3 - y^3$. Classify each of them as a local maximum, local minimum, or neither. (optional) For a variety of values of c, including the critical values, use a computer to draw the curve $f(x,y) = c$.

(8) (*) Let $f(x,y,z) = x^3 + y^3 + z^3 - 3(x+y+z)$. Find all critical points of f. Find a local maximum of f. (optional) For a variety of values of c, including the critical values, use a computer to draw the surface $f(x,y,z) = c$.

(9) Find all first and second partial derivatives of $f(x,y,z) = x^3 + y^2 z + xz - xy^3$.

(10) (*) Find all $c \in \mathbb{R}$ such that there is a differentiable function $f(x,y)$ with $D_1 f(x,y) = x^3 - 4xy$ and $D_2 f(x,y) = y^2 - cx^2$. For each possible value of c, find all possible functions f.

(11) Define $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2} \text{ if } (x,y) \neq (0,0) \\ 0 \text{ if } (x,y) = (0,0) \end{cases}.$$

  (a) Show that f is differentiable at 0 (and hence everywhere).
  (b) Find $D_2 f(x,0)$ and $D_1 f(0,y)$.
  (c) Show that $D_{12} f(0,0) \neq D_{21} f(0,0)$.