MATH 159 – HOMEWORK 5

Due 2pm on May 4th. This week, everything is graded.

(1) Prove that the absolute value function $\mathbb{R}^n \to \mathbb{R}$ taking x to $|x|$ is continuous.
(2) Prove directly from the definition that the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x + y$ is continuous.
(3) Prove that, if $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^l$ are continuous, then so is $g \circ f$.
(4) Prove that, if $f : \mathbb{R}^d \to \mathbb{R}^e$ is continuous and $x_n \to x \in \mathbb{R}^d$, then $f(x_n) \to f(x)$.
(5) If $f : \mathbb{R}^n \to \mathbb{R}^m$ is a function, with $f(x) = (f^1(x), \ldots, f^m(x))$, then f is continuous if and only if each $f^i : \mathbb{R}^n \to \mathbb{R}$ is continuous.
(6) Say, with proof, whether the following sets are open, closed, or neither. Draw pictures where you can.
  (a) $\{(x, y, z) \in \mathbb{R}^3 : |x| + |y| + |z| < 1\}$;
  (b) $\{x \in \mathbb{R}^n : |x| = 1\}$;
  (c) $\{(x, y) \in \mathbb{R}^2 : x + y \in \mathbb{Q}\}$;
  (d) $\{(x, y) \in \mathbb{R}^2 : y = 0, x > 0\}$.
(7) Find a closed subset of $\mathbb{R}^2$ whose projection onto the x-axis is not closed.
(8) Prove that the product xyz takes a maximum value subject to the constraints $0 \le x, y, z \le 1$, $x + y + z = 1$. (optional) Find this maximum?