MATH 163 – HOMEWORK 4

Due noon on May 1st.

(1) Spivak 1-1.
(2) Spivak 1-4.
(3) Spivak 1-5.
(4) By applying the Cauchy–Schwarz inequality to (x, y, z) and $(1, 1, 1)$, show that
$$(x + y + z)^2 \leq 3(x^2 + y^2 + z^2)$$
for any $(x, y, z) \in \mathbb{R}^3$.
(5) For each of the following pairs of vectors x, y, find the unit vector $\hat{y}$, the projection of x on y, and the angle between x and y:
 (a) $x = (1, 1)$, $y = (0, 2)$;
 (b) $x = (1, -1, 0)$, $y = (1, 0, -1)$;
 (c) $x = (1, 1, 1, 1)$, $y = (2, 0, 1, -3)$.
(6) Prove the cosine rule (if x and y are vectors and θ is the angle between them, then $|x - y|^2 = |x|^2 + |y|^2 - 2|x||y|\cos\theta$).
(7) (a) Find a vector perpendicular to $(1, 3)$;
 (b) Find a vector perpendicular to $(1, 1, -2)$ and $(1, -1, 0)$;
 (c) Find two non-parallel vectors perpendicular to $(1, 2, 3)$;
 (d) Find two non-parallel vectors perpendicular to $(1, 1, 1, 1)$ and $(1, 2, 3, 4)$.
(8) Prove directly from the definition that $s : \mathbb{R}^2 \to \mathbb{R}$ given by $s(x+y) = x+y$ is continuous.
(9) Say, with proof, whether the following subsets of $\mathbb{R}^n$ are open, closed, or neither. Draw pictures where you can.
 (a) $\{(x, y, z) \in \mathbb{R}^3 : |x| + |y| + |z| < 1\}$;
 (b) $\{x \in \mathbb{R}^n : |x| = 1\}$;
 (c) $\{(x, y) \in \mathbb{R}^2 : x + y \in \mathbb{Q}\}$;
 (d) $\{(x, y) \in \mathbb{R}^2 : y = 0, x > 0\}$.
(10) Spivak 1-19.
(11) Prove that $\{x \in \mathbb{Q} : x < \sqrt{2}\}$ is both an open and closed subset of $\mathbb{Q}$.
(12) Suppose that U is a subset of $\mathbb{R}$ that is both open and closed. Show that the function $f : \mathbb{R} \to \mathbb{R}$ given by
$$f(x) = \begin{cases} 1 & \text{if } x \in U \\ 0 & \text{if } x \notin U \end{cases}$$
is continuous. Using the intermediate value theorem, deduce that either U is empty or $U = \mathbb{R}$.
(13) Find a closed subset of $\mathbb{R}^2$ whose projection onto the x-axis (that is, its image under the map π_1) is not closed.