MATH 159 – HOMEWORK 4

Due 1pm on October 28th. This week, all problems are graded.

(1) Show that, if (a, b) and (c, d) are bounded open intervals in $\mathbb{R}$ with non-empty intersection, then their union is also a bounded open interval.

(2) Write $(0, \infty)$ as a union of countably many closed, bounded intervals.

(3) For $n \in \mathbb{N}$, let $a_n = 1 - \frac{1}{n}$ and $b_n = 1 + \frac{1}{n}$. Prove that

$$\bigcap_{n \in \mathbb{N}} [a_n, b_n] = \{1\}.$$

(4) Show that, if $a, b \in \mathbb{R}$, then $|ab| = |a||b|$.

(5) Show that, if $a, b \in \mathbb{R}$, then $|a - b| \geq ||a| - |b||$.

(6) Suppose that $(a_n)_n$ and $(b_n)_n$ are convergent sequences of real numbers with

$$\lim_{n \to \infty} a_n = a$$

and

$$\lim_{n \to \infty} b_n = b.$$

Prove that:

(a) if $c_n = a_n - b_n$, then

$$\lim_{n \to \infty} c_n = a - b;$$

(b) if $a_n > b_n$ for all n, then $a \geq b$;

(c) it need not be that case that, if $a_n > b_n$ for all n, then $a > b$.